软件分析与验证前沿

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Outline

- First example: Available expressions
- Basic principles
- More examples ←
- Solving data flow problems
- Inter-procedural analysis
- Sensitivities

Data Flow Analyses

- **Seen previously**
  - Available expressions
- **Next**
  - Reaching definitions
  - Very busy expressions
  - Live variables

Reaching Definitions Analysis

Goal: For each program point, compute which assignments may have been made and may not have been overwritten

- Useful in various program analyses
- E.g., to compute a data flow graph

A reaching definition for a given instruction is an earlier instruction whose ***target variable*** can reach (be assigned to) the given one without an intervening assignment.

https://en.wikipedia.org/wiki/Reaching_definition

Example

```
var x = 5;
var y = 1;
while(x > 1) {
  y = x * y;
  x = x - 1;
}
```

A reaching definition for a given instruction is an earlier instruction whose ***target variable*** can reach (be assigned to) the given one without an intervening assignment.

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Example

```
var x = 5;
var y = 1;
while(x > 1) {
  y = x * y;
  x = x - 1;
}
```

Definition reaches entry of this statement

A reaching definition for a given instruction is an earlier instruction whose ***target variable*** can reach (be assigned to) the given one without an intervening assignment.

https://en.wikipedia.org/wiki/Reaching_definition

Example

```
var x = 5;
var y = 1;
while(x > 1) {
  y = x * y;
  x = x - 1;
}
```

All definitions reach the entry of this statement

A reaching definition for a given instruction is an earlier instruction whose ***target variable*** can reach (be assigned to) the given one without an intervening assignment.

https://en.wikipedia.org/wiki/Reaching_definition

Example

```
var x = 5;
var y = 1;
while(x > 1) {
  y = x * y;
  x = x - 1;
}
```

Three definitions reach entry of this statement

A reaching definition for a given instruction is an earlier instruction whose ***target variable*** can reach (be assigned to) the given one without an intervening assignment.

https://en.wikipedia.org/wiki/Reaching_definition

Defining the Analysis

- **Domain: Definitions (assignments) in the code**
  - Set of pairs (v, s) of variables and statements
  - (v, s) means a definition of v at s
- **Direction: Forward**
- **Meet operator: Union**
  - Because we care about definitions that *may* reach a program point

Defining the Analysis (2)

- **Transfer function**:

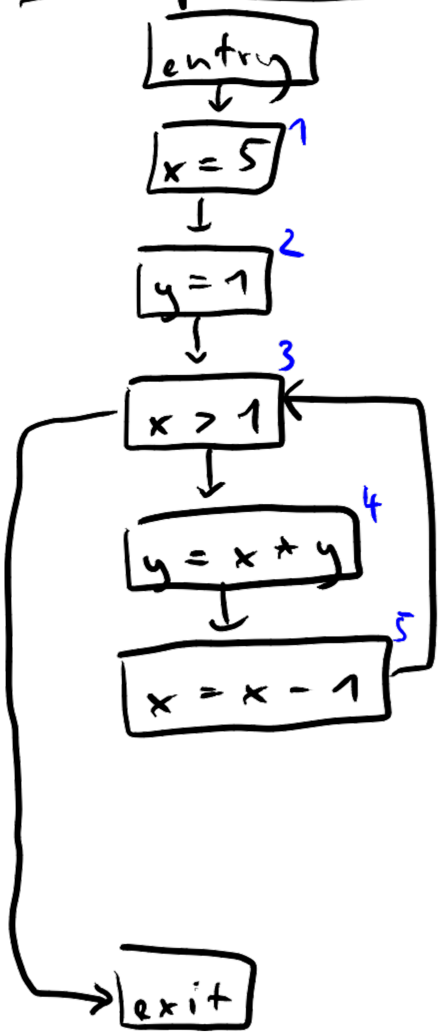
  $RD_{exit}(s) = (RD_{entry}(s) \setminus kill(s)) \cup gen(S)$

- **Function** $gen(s)$
  - If s is assignment to v: (v, s)
  - Otherwise: Empty set
- **Function** $kill(s)$
  - If s is assignment to v: (v, s') for all s' that define v
  - Otherwise: Empty set

Defining the Analysis (3)

- **Boundary condition: Entry node starts with all variables undefined**
  - Special "statement" for undefined variables: ?
  - $RD_{entry}(entryNode) = \{(v, ?) \mid v \in Vars\}$
- **Initially, all nodes have no reaching definitions**

Example: Reaching Definitions



s	gen(s)	kill(s)
1	{(x, 1)}	{(x, 1), (x, 5), (x, ?)}
2	{(y, 2)}	{(y, ?), (y, 2), (y, 4)}
3	∅	∅
4	{(y, 4)}	{(y, 2), (y, 4), (y, ?)}
5	{(x, 5)}	{(x, 1), (x, 5), (x, ?)}

Data Flow Equations

$RD_{entry}(1) = \{(x,?), (y,?)\}$

$RD_{entry}(2) = RD_{exit}(1)$

$RD_{entry}(3) = RD_{exit}(2) \cup RD_{exit}(5)$

$RD_{entry}(4) = RD_{exit}(3)$

$RD_{entry}(5) = RD_{exit}(4)$

$RD_{exit}(1) = (RD_{entry}(1) \setminus \{(x,1), (x,5), (x,?)\}) \cup \{(x,1)\}$

$RD_{exit}(2) = (RD_{entry}(2) \setminus \{(y,2), (y,4), (y,?)\}) \cup \{(y,2)\}$

$RD_{exit}(3) = RD_{entry}(3)$

$RD_{exit}(4) = (RD_{entry}(4) \setminus \{(y,2), (y,4), (y,?)\}) \cup \{(y,4)\}$

$RD_{exit}(5) = (RD_{entry}(5) \setminus \{(x,1), (x,5), (x,?)\}) \cup \{(x,5)\}$

Solution

s	$RD_{entry}(s)$	$RD_{exit}(s)$
1	$\{(x,?), (y,?)\}$	...
2	...	...
3	$\{(x,1), (y,2), (y,4), (x,5)\}$	...
4	...	...
5	...	$\{(y,4), (x,5)\}$

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Very Busy Expression Analysis

Goal: For each program point, find expressions that must be very busy

- "Very busy": On all future paths, expression will be used before any of the variables in it are redefined
- Useful for program optimizations, e.g., hoisting
  - Hoisting an expression: Pre-compute it, e.g., before entering a block, for later use

An expression is very busy at *p* if it is evaluated on *every* path *from p* before it changes in value.

Example

```
if(a > b) {
  x = b - a;
  y = a - b;
}else{
  y = b - a;
  x = a - b;
}
```

Example

```
if (a > b) {
  x = b - a;
  y = a - b;
}else{
  y = b - a;
  x = a - b;
}
```

a - b and b - a are very busy here

Defining the Analysis

- **Domain: All non-trivial expressions occurring in the code**
- **Direction: Backward**
- **Meet operator: Intersection**
  - Because we care about very busy expressions that *must* be used

Defining the Analysis (2)

Transfer function:

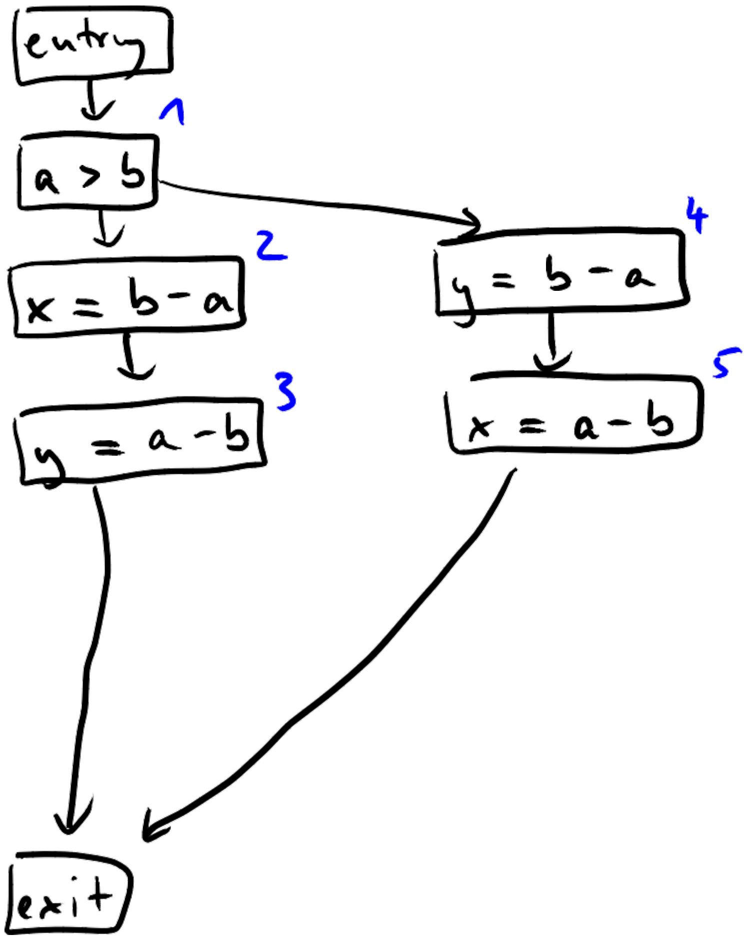
$$VB_{entry}(s) = (VB_{exit}(s) \setminus kill(s)) \cup gen(S)$$

- Backward analysis: Returns expressions that are very busy expressions at entry of statement
- Function $gen(s)$
  - All expressions e that appear in s
- Function $kill(s)$
  - If s assigns to x, all expressions in which x occurs
  - Otherwise: Empty set

Defining the Analysis (3)

- **Boundary condition: Final node starts with no very busy expressions**
  - $VB_{exit}(finalNode) = \emptyset$
- **Initially, all nodes have no very busy expressions**

Example: Very Busy Expressions Analysis



s	gen(s)	kill(s)
1	{a > b}	∅
2	{b - a}	∅
3	{a - b}	∅
4	{b - a}	∅
5	{a - b}	∅

s	$VB_{entry}(s)$	$VB_{exit}(s)$
1	{a-b, b-a, a>b}	{a-b, b-a}
2	{a-b, b-a}	{a-b}
3	{a-b}	∅
4	{a-b, b-a}	{a-b}
5	{a-b}	∅

Live Variables Analysis

Goal: For each statement, find variables that are may be “live” at the exit from the statement

- ”Live”: The variable is used before being redefined
- Useful, e.g., for identifying dead code
  - Bug detection: Dead assignments are typically unintended
  - Optimization: Remove dead code

A variable is live at some point if it holds a value that may be needed in the future, or equivalently if its value may be read before the next time the variable is written to.

https://en.wikipedia.org/wiki/Live-variable_analysis

Example

```
x =2;
y =4;
x =1;
if(y > x) {
  z =y;
}else{
  z = y * y;
  x =z;
}
```

Example

```
x =2;
y =4;
x =1;
if(y > x) {
  z =y;
}else{
  z = y * y;
  x =z;
}
```

x is not live after this statement

Example

```
x =2;
y =4;
x =1;
if(y > x) {
  z =y;
}else{
  z = y * y;
  x =z;
}
```

Both x and y are live after this statement

Defining the Analysis

- **Domain: All variables occurring in the code**
- **Direction: Backward**
- **Meet operator: Union**
  - Because we care about whether a variable *may* be used

Defining the Analysis (2)

Transfer function:

$$LV_{entry}(s) = (LV_{exit}(s) \setminus kill(s)) \cup gen(S)$$

- Backward analysis: Returns set of variables that are live at entry of statement
- Function *gen*(*s*)
  - All variables *v* that are used in *s*
- Function *kill*(*s*)
  - If *s* assigns to *x*, then it kills *x*
  - Otherwise: Empty set

Defining the Analysis (3)

- **Boundary condition: Final node starts with no live variables**
  - $LV_{exit}(finalNode) = \emptyset$
- **Initially, all nodes have no live variables**

Quiz: Live Variables

```
x =2;
y =4;
x =1;
if(y>x){
   z =y;
}else{
   z=y*y;
   x =z;
}
```

Compute the live variables before and after every statement.

Quiz: Live Variables

```
x =2;
y =4;
x =1;
if(y>x){
   z =y;
}else{
   z=y*y;
   x =z;
}
```

Compute the live variables before and after every statement.

Compute:

(1) gen(s) and kill(s)

(2) $LV_{entry}(s)$ and $LV_{exit}(s)$

Live variable analysis: Example

s	$LV_{entry}(s)$	$LV_{exit}(s)$
1	$\emptyset$	$\emptyset$
2	$\emptyset$	$\{y\}$
3	$\{y\}$	$\{x, y\}$
4	$\{x, y\}$	$\{y\}$
5	$\{y\}$	$\emptyset$
6	$\{y\}$	$\{z\}$
7	$\{z\}$	$\emptyset$

$$LV_{exit}(4) = LV_{entry}(5) \cup LV_{entry}(6)$$

Applications of Four Analyses

- **Available Expressions**: Optimization
  - don't recompute expressions that are still available

- **Very Busy Expressions**: Optimization
  - move expression to a common program point

- **Reaching Definitions**: Bug-finding and Optimization
  - uninitialized variables, constant propagation

- **Live Variables**: Optimization
  - don't store variables that aren't live, eliminate assignments where variables are dead