

# 软件分析与验证前沿

苏亭

软件科学与技术系

# Big Picture

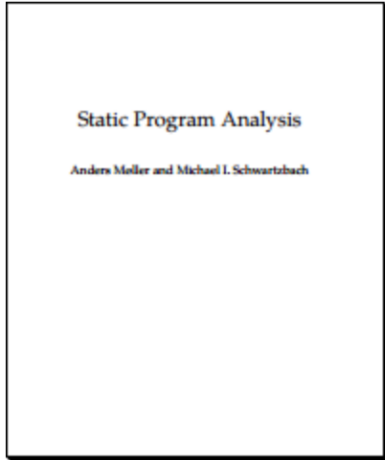
---

- Introduction to static analysis
- Many ways of **formulating and implementing analyses**
- One popular way of formulating a static analysis: **Data flow analysis**

# Big Picture

---

- **Introduction to static analysis**
- Many ways of formulating and implementing analyses
- One popular way of formulating a static analysis: Data flow analysis



# Static Program Analysis

[Anders Møller](#) and Michael I. Schwartzbach  
Department of Computer Science, Aarhus University

Last revision: November 2022



[PDF](#)



[BibTeX](#)

Static program analysis is the art of reasoning about the behavior of computer programs without actually running them. This is useful not only in optimizing compilers for producing efficient code but also for automatic error detection and other tools that can help programmers.

As known from Turing and Rice, all interesting properties of the behavior of programs written in common programming languages are mathematically undecidable. This means that automated reasoning of software generally must involve approximation. It is also well known that testing may reveal errors but not show their absence. In contrast, static program analysis can - with the right kind of approximations - check all possible executions of the programs and provide guarantees about their properties. The challenge when developing such analyses is how to ensure high precision and efficiency to be practically useful.

This teaching material concisely presents the essential principles and algorithms for static program analysis. We emphasize a constraint-based approach where suitable constraint systems conceptually divide analysis into a front-end that generates constraints from program code and a back-end that solves the constraints to produce the analysis results. The style of presentation is intended to be precise but not overly formal. The readers are assumed to be familiar with advanced programming language concepts and the basics of compiler construction.

The concepts are explained using a tiny imperative programming language, TIP, which suffices to illustrate the main challenges that arise with mainstream languages.

The lecture notes, slides, implementation, and exercises have been developed since 2008 for our graduate-level course at Aarhus University. We continue to update the material regularly. Suggestions for improvements are welcome!

课程资料: <http://cs.au.dk/~amoeller/spa/>

# Questions about programs

---

- Does the program terminate on all inputs?
- How large can the heap become during execution?
- Can sensitive information leak to non-trusted users?
- Can non-trusted users affect sensitive information?
- Are buffer-overruns possible?
- Data races?
- SQL injections?
- Cross-Site Scripting (XSS)?
- ...

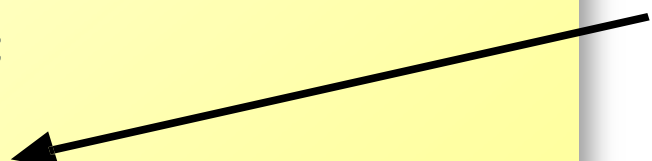


# Program points

---

```
foo(p,x) {  
    var f,q;  
    if (*p==0) { f=1; }  
    else {  
        q = alloc 10;  
        *q = (*p)-1;  
        f=(*p)*(x(q,x));  
    }  
    return f;  
}
```

any point in the program  
= any value of the PC



## Invariants:

A property holds at a program point if it holds in any such state for any execution with any input

# Questions about program points

---

- Will the value of  $x$  be read in the future?
- Can the pointer  $p$  be null?
- Which variables can  $p$  point to?
- Is the variable  $x$  initialized before it is read?
- What is a lower and upper bound on the value of the integer variable  $x$ ?
- At which program points could  $x$  be assigned its current value?
- Do  $p$  and  $q$  point to disjoint structures in the heap?
- Can this assert statement fail?

# Why are the answers interesting?

---

- Increase efficiency
  - resource usage
  - compiler optimizations
- Ensure correctness
  - verify behavior
  - catch bugs early
- Support program understanding
- Enable refactorings





# Testing?

---

*“Program testing can be used to show the presence of bugs, but never to show their absence.”*

[Dijkstra, 1972]

Nevertheless, testing often takes 50% of the development cost

# Programs that reason about programs

---



# Requirements to the perfect program analyzer



**SOUNDNESS** (don't miss any errors)



**COMPLETENESS** (don't raise false alarms)



**TERMINATION** (always give an answer)

# Rice's theorem, 1953

## CLASSES OF RECURSIVELY ENUMERABLE SETS AND THEIR DECISION PROBLEMS<sup>(1)</sup>

BY  
H. G. RICE

**1. Introduction.** In this paper we consider classes whose elements are recursively enumerable sets of non-negative integers. No discussion of recursively enumerable sets can avoid the use of such classes, so that it seems desirable to know some of their properties. We give our attention here to the properties of complete recursive enumerability and complete recursiveness (which may be intuitively interpreted as decidability). Perhaps our most interesting result (and the one which gives this paper its name) is the fact that no nontrivial class is completely recursive.

We assume familiarity with a paper of Kleene [5]<sup>(2)</sup>, and with ideas which are well summarized in the first sections of a paper of Post [7].

### I. FUNDAMENTAL DEFINITIONS

**2. Partial recursive functions.** We shall characterize recursively enumer-



COROLLARY B. *There are no nontrivial c.r. classes by the strong definition.*

# Rice's theorem

---

Any non-trivial property of the behavior of programs in a Turing-complete language is undecidable!



# Approximation

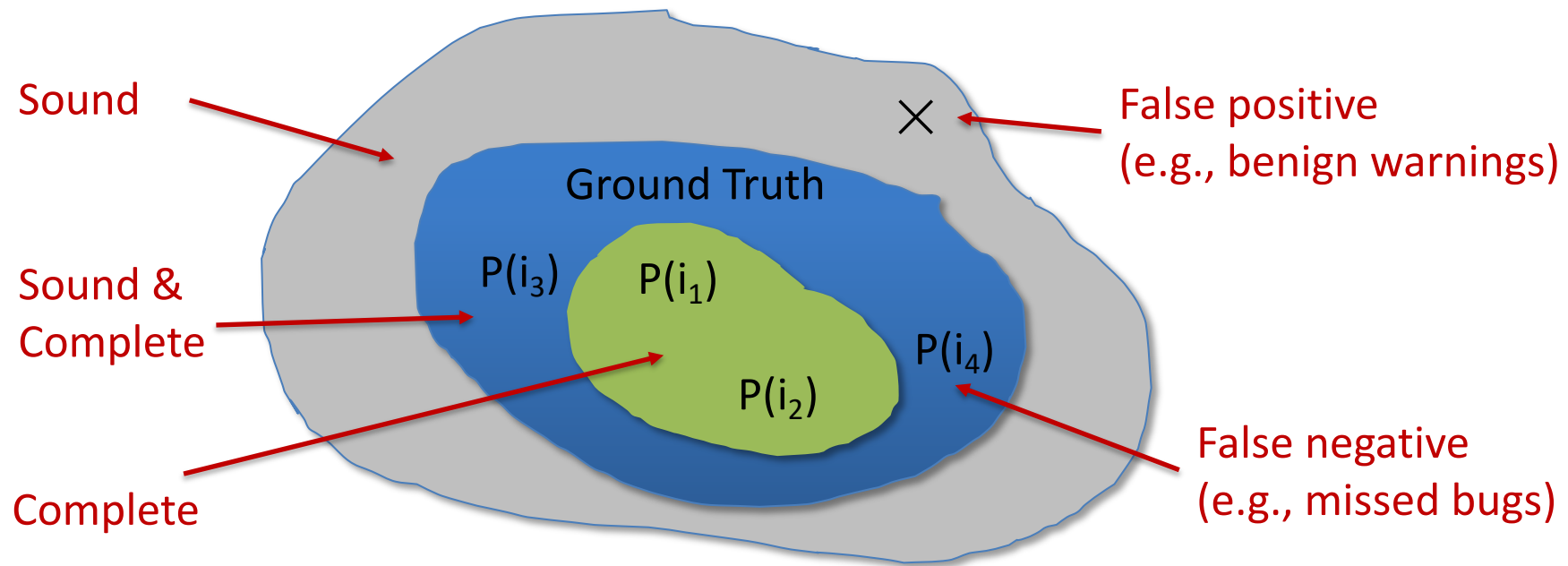
---

- *Approximate* answers may be decidable!

# Soundness & Completeness

---

Program  $P$ , Input  $i$ , Behavior  $P(i)$



All possible behaviors (what we want, ideally)



Under-approximation (e.g., testing, dynamic analysis)



Over-approximation (most static analysis)

# Approximation

---

- *Approximate* answers may be decidable!



# Approximation

---

- *Approximate* answers may be decidable!
- The approximation must be *conservative*:
  - i.e. only err on “the safe side”
  - which direction depends on the *client application*
- We'll focus on decision problems
- More subtle approximations if not only “*yes*”/“*no*”
  - e.g. memory usage, pointer targets

# Example approximations

---

- Decide if a given function is ever called at runtime:
  - if “*no*”, remove the function from the code
  - if “*yes*”, don’t do anything
  - the “*no*” answer *must* always be correct if given
- Decide if a cast  $(A)x$  will always succeed:
  - if “*yes*”, don’t generate a runtime check
  - if “*no*”, generate code for the cast
  - the “*yes*” answer *must* always be correct if given

# Beyond “yes”/“no” problems

---

- How much memory / time may be used in any execution?
- Which variables may be the targets of a pointer variable  $p$ ?

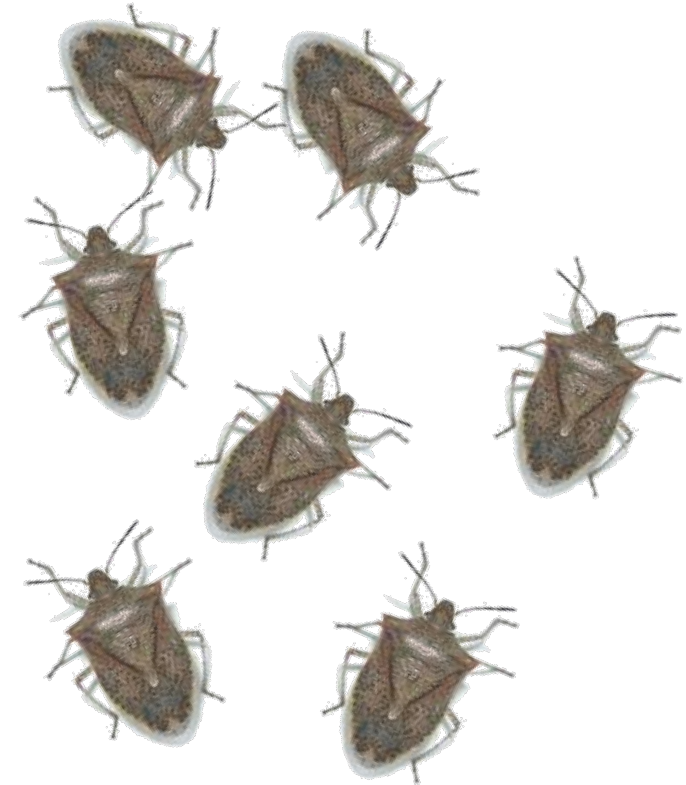
# The engineering challenge

---

- A correct but trivial approximation algorithm may just give the useless answer every time
- The *engineering challenge* is to give the useful answer often enough to fuel the client application
- ... and to do so within reasonable time and space
- This is the hard (and fun) part of static analysis!

# Bug finding

```
1  int main() {  
2      char *p,*q;  
3      p = NULL;  
4      printf("%s",p);  
5      q = (char *)malloc(100);  
6      p = q;  
7      free(q);  
8      *p = 'x';  
9      free(p);  
10     p = (char*)malloc(100);  
11     q = (char*)malloc(100);  
12     q = p;  
13     strcat(p,q);  
14 }
```



```
gcc -Wall foo.c  
lint foo.c
```

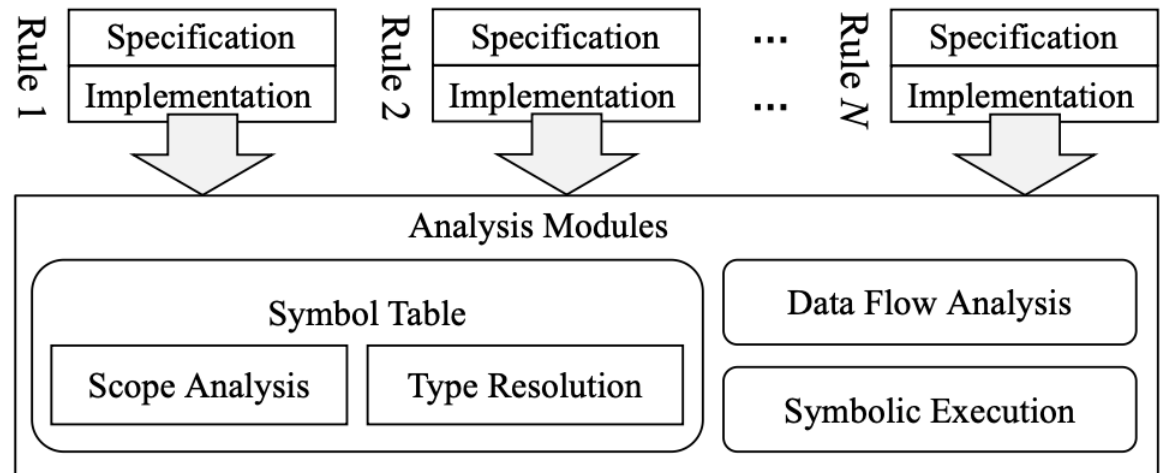
**No errors!**



# Static Code Analyzers (Linters)

---

PMD - source code analyzer



PMD: <https://github.com/pmd/pmd>

SpotBugs: <https://github.com/spotbugs/spotbugs>

SonarQube: <https://github.com/SonarSource/sonar-java>






# Does anyone use static program analysis?

---

For optimization:

- every optimizing compiler and modern JIT

For verification or error detection:

- **Astrée**
- 
- **COVERITY**
- 
- 
- 
- 

# software development

POSTED ON SEP 6, 2017 TO [ANDROID](#), [DEVELOPER TOOLS](#), [IOS](#)

## Finding interprocedural bugs Infer static analysis



SAM BLACKSHEAR

The capabilities of static analysis tools like [Infer](#) and [FindBugs](#) are growing. Our work on the [Infer static analysis](#) source analysis tools like [FindBugs](#) procedural bugs, or bugs that

We'll take a look at two examples of interprocedural bugs found in source code. DuckDuckGo Android app. In the tools mentioned above, we use [Infer](#) Analyzer — only intra-file and unit, a file-with-includes).

Inter-procedural bugs are significant. Facebook developers have found that they can have a large impact; we found them in Facebook. As we have found in other codebases that consist of millions of lines of code and undergo thousands of modifications per day.



WIRED

How Facebook Catches Bugs in Its 100 Million Lines of Code

BUSINESS

CULTURE

GEAR

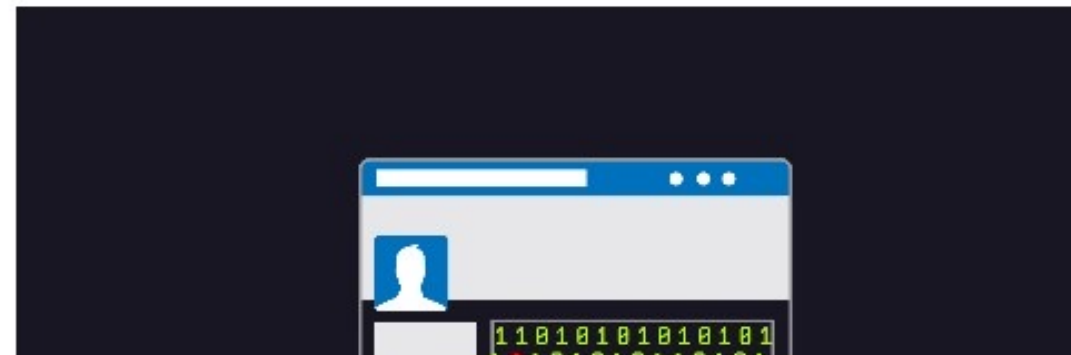
IDEAS

SHARE



LILY HAY NEWMAN SECURITY 08.15.19 05:03 PM

## HOW FACEBOOK CATCHES BUGS IN ITS 100 MILLION LINES OF CODE





# Big Picture

---

- **Introduction to static analysis**
- Many ways of formulating and implementing analyses
- One popular way of formulating a static analysis: Data flow analysis

# Big Picture

---

- Introduction to static analysis
- **Many ways of formulating and implementing analyses**
- **One popular way of formulating a static analysis: Data flow analysis**

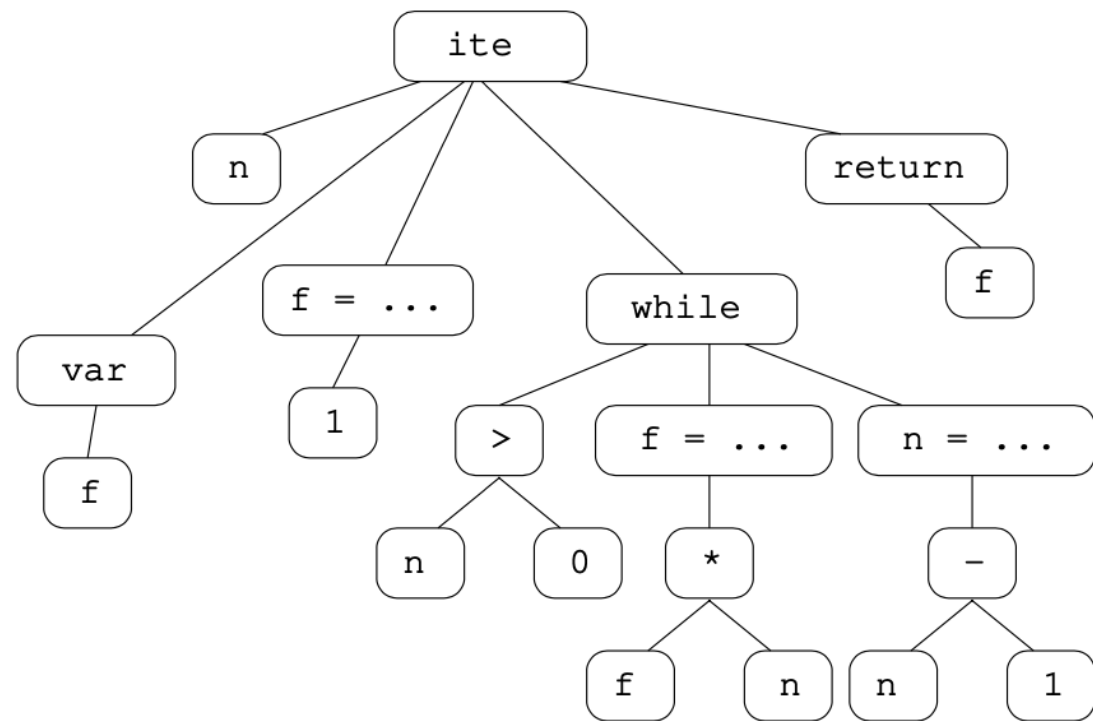
# How Programs are Represented?

---

# Abstract Syntax Trees

---

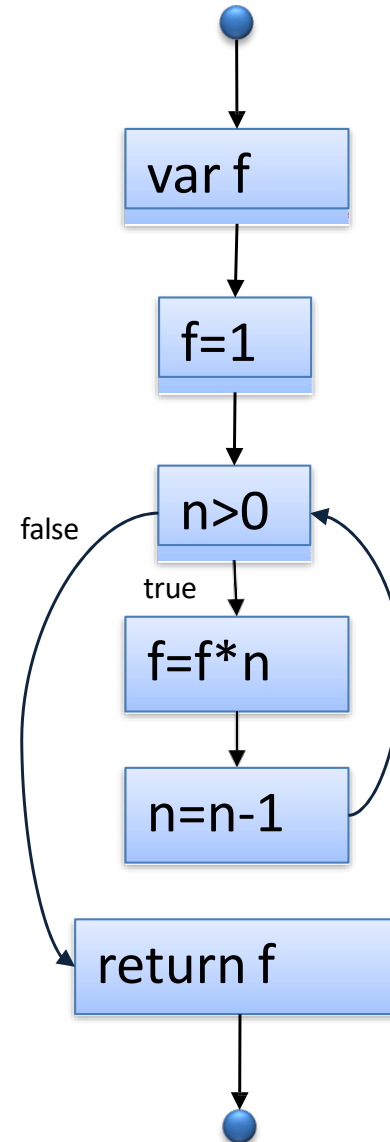
```
ite(n) {  
  var f;  
  f = 1;  
  while (n>0) {  
    f = f*n;  
    n = n-1;  
  }  
  return f;  
}
```



# Control flow graphs

---

```
ite(n) {  
  var f;  
  f = 1;  
  while (n>0) {  
    f = f*n;  
    n = n-1;  
  }  
  return f;  
}
```



# Control flow graphs

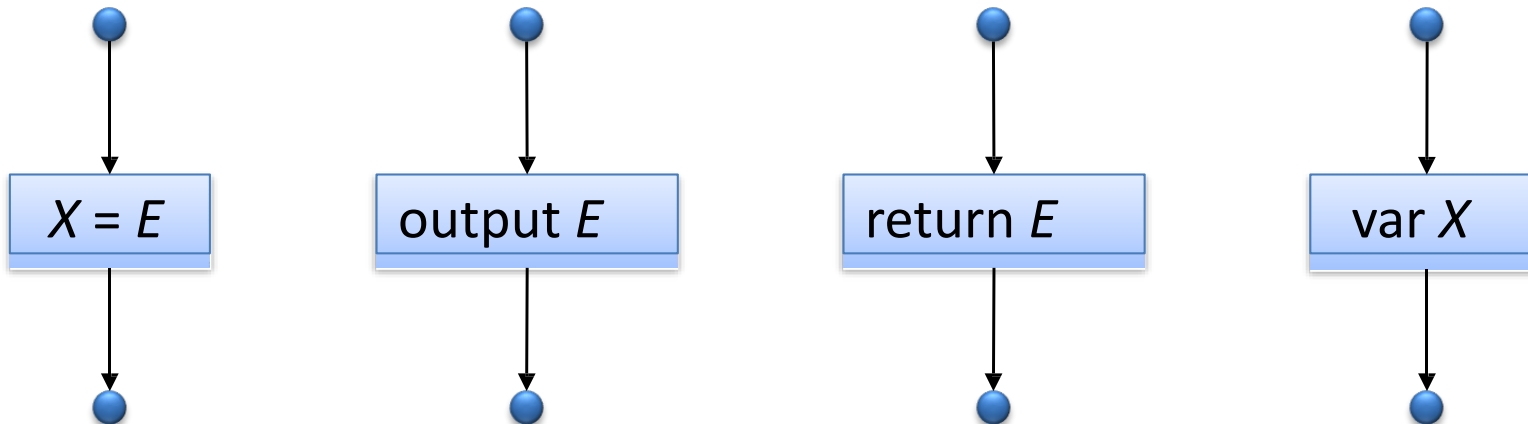
---

- A *control flow graph* (CFG) is a directed graph:
  - *nodes* correspond to program points  
(either immediately before or after statements)
  - *edges* represent possible flow of control
- A CFG always has
  - a single point of *entry*
  - a single point of *exit*  
(think of them as no-op statements)
- Let  $v$  be a node in a CFG
  - $pred(v)$  is the set of predecessor nodes
  - $succ(v)$  is the set of successor nodes

# CFG construction (1/3)

---

- CFGs are constructed inductively
- CFGs for simple statements etc.:

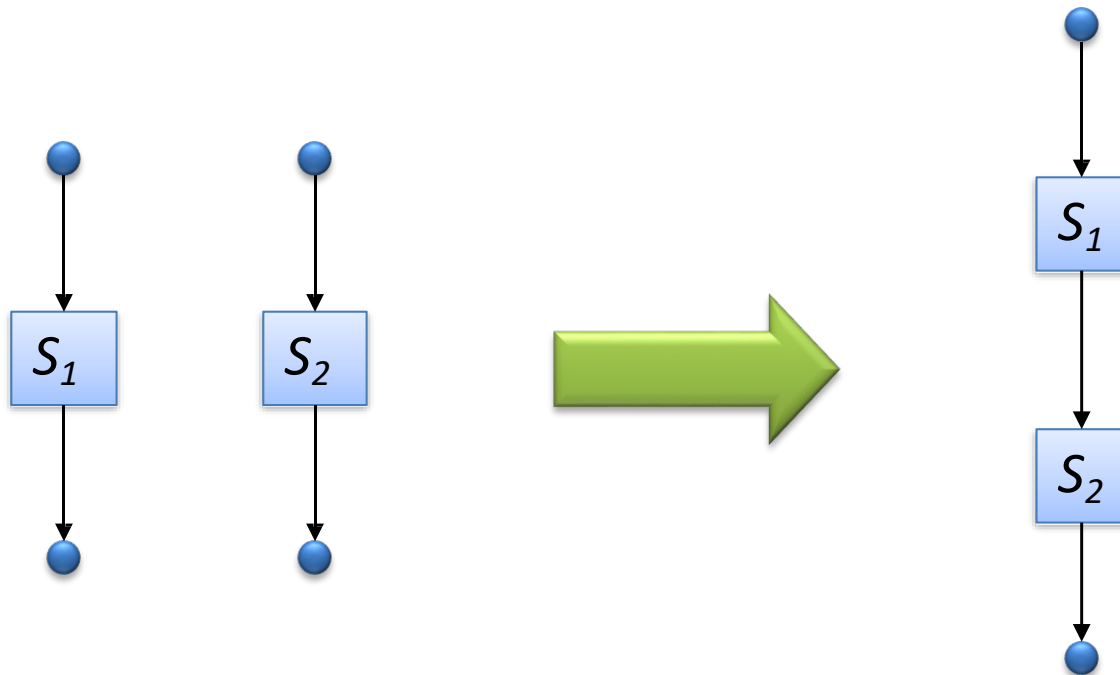


# CFG construction (2/3)

---

For a statement sequence  $S_1 S_2$ :

- eliminate the exit node of  $S_1$  and the entry node of  $S_2$
- glue the statements together

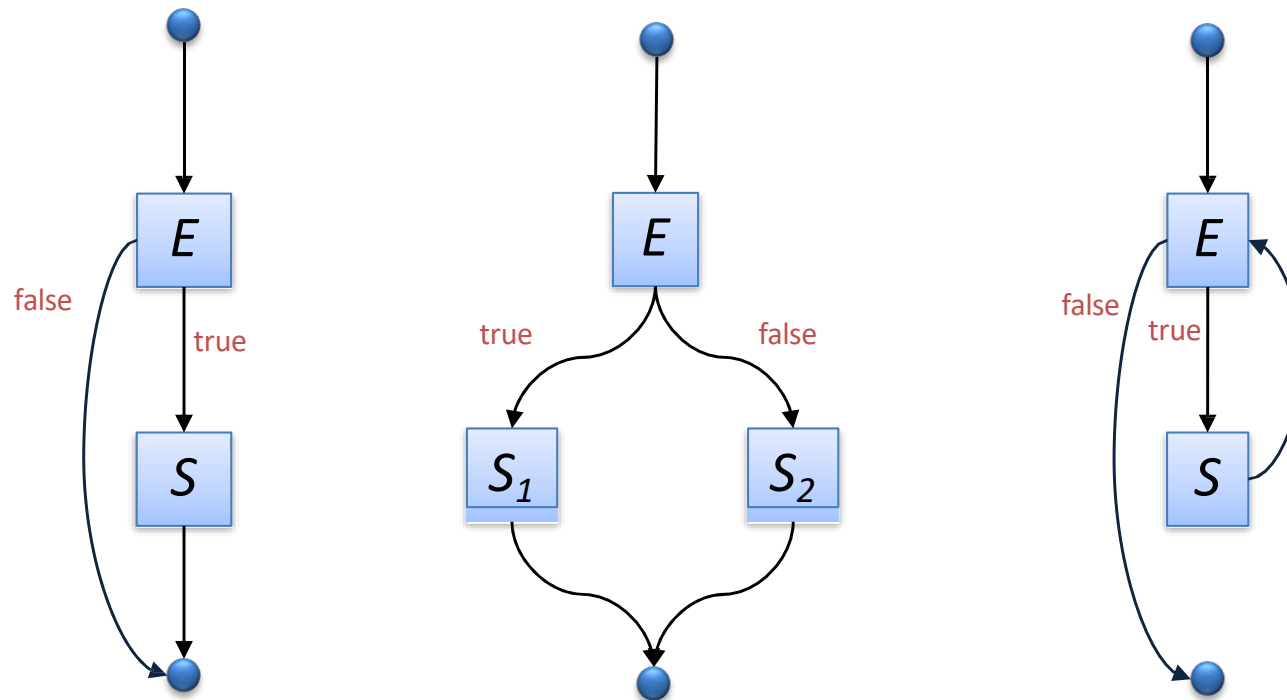




# CFG construction (3/3)

---

Similarly for the other control structures:



# Normalization

---

- Sometimes convenient to ensure that each CFG node performs only one operation
- *Normalization (A-normal form)*: flatten nested expressions, using fresh variables

```
x = f(y+3)*5;
```



```
t1 = y+3;  
t2 = f(t1);  
x = t2*5;
```

# Data Flow Analysis

---

## Basic idea

- Propagate analysis information along the edges of a control flow graph
- Goal: Compute analysis state at each program point
- For each statement, define how it affects the analysis state
- For loops: Iterate until fix-point reached

# Outline

---

- **First example: Available expressions** ←
- **Basic principles**
- **More examples**
- **Solving data flow problems**
- **Inter-procedural analysis**
- **Sensitivities**

# Available Expression Analysis

---

**Goal:** For each program point, compute which **expressions must have already been computed**, and not later modified

- Useful, e.g., to **avoid re-computing** an expression
- Used as part of **compiler optimizations**

To be available on a program point, the **operands** of the expression should not be modified on **any path** from the occurrence of that expression to the program point.

[https://en.wikipedia.org/wiki/Available\\_expression](https://en.wikipedia.org/wiki/Available_expression)

# Example

---

```
var x = a + b;  
var y = a * b;  
while (y > a + b) {  
    a = a - 1;  
    x = a + b;  
}
```

To be available on a program point, the **operands** of the expression should not be modified on **any path** from the occurrence of that expression to the program point.

[https://en.wikipedia.org/wiki/Available\\_expression](https://en.wikipedia.org/wiki/Available_expression)

# Example

---

```
var x = a + b;  
var y = a * b;  
while (y > a + b) {  
    a = a - 1;  
    x = a + b;  
}
```

Available every time  
execution reaches  
this point

To be available on a program point, the **operands** of the expression should not be modified on **any path** from the occurrence of that expression to the program point.

[https://en.wikipedia.org/wiki/Available\\_expression](https://en.wikipedia.org/wiki/Available_expression)

# Transfer Functions

---

- **Transfer function of a statement:**  
**How the statement affects the analysis state**
  - Q Here: Analysis state = available expressions
- **Two functions**
  - Q **gen**: Available expressions **generated** by a statement
  - Q **kill**: Available expressions **killed** by a statement



# *gen* Function

---

**Function**  $gen : Stmt \rightarrow P(Expr)$

- A statement **generates an available expressions**  $e$  if
  - Q it evaluates  $e$  and
  - Q it **does not** later write any variable used in  $e$
- Otherwise, function returns **empty set**

**Example:**

**`var x = a * b;` generates `a * b`**

# *kill* Function

---

**Function**  $kill : Stmt \rightarrow P(Expr)$

- A statement **kills an available expressions**  $e$  if
  - Q it modifies any of the variables used in  $e$
- Otherwise, function returns **empty set**

**Example:**

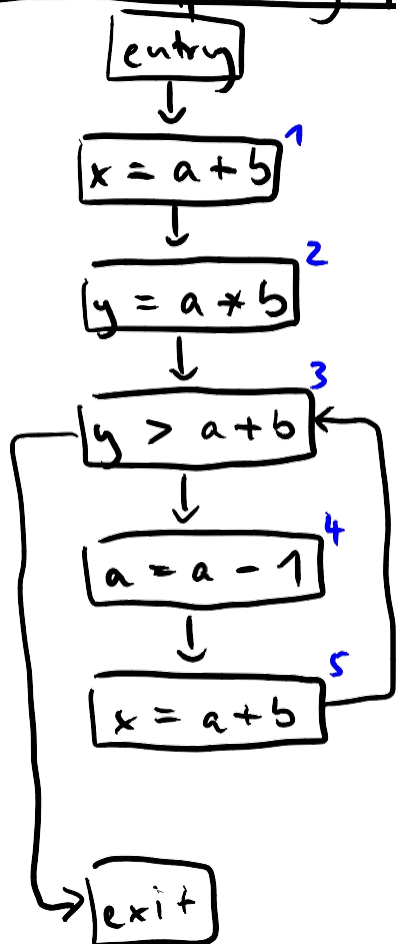
**$a = 23;$  kills  $a * b$**

# Example

---

```
var x = a + b;  
var y = a * b;  
while (y > a + b) {  
    a = a - 1;  
    x = a + b;  
}
```

## Control flow graph



## Non-trivial expressions

$a + b$   
 $a * b$   
 $a - 1$

Transfer function for each statement:

Statement $s$	$gen(s)$	$k:U(s)$
1	$\{a + b\}$	$\emptyset$
2	$\{a * b\}$	$\emptyset$
3	$\{a + b\}$	$\emptyset$
4	$\emptyset$	$\{a - 1, a + b, a * b\}$
5	$\{a + b\}$	$\emptyset$

# Propagating Available Expressions

---

- Initially, no available expressions
- **Forward analysis**: Propagate available expressions in the direction of control flow
- For each statement  $s$ , **outgoing available expressions** are:  
 $\text{incoming avail. exprs.} \text{ minus } kill(s) \text{ plus } gen(s)$
- When **control flow splits**, propagate available expressions **both ways**
- When **control flows merge**, **intersect** the incoming available expressions

### Data flow equations

$AE_{\text{entry}}(s)$  ... avail. express. at entry of  $s$

$AE_{\text{exit}}(s)$  ... avail. express. at exit of  $s$

$$AE_{\text{entry}}(1) = \emptyset$$

$$AE_{\text{entry}}(2) = AE_{\text{exit}}(1)$$

$$AE_{\text{entry}}(3) = AE_{\text{exit}}(2) \cap AE_{\text{exit}}(5)$$

$$AE_{\text{entry}}(4) = AE_{\text{exit}}(3)$$

$$AE_{\text{entry}}(5) = AE_{\text{exit}}(4)$$

$$AE_{\text{exit}}(1) = AE_{\text{entry}}(1) \cup \{a+b\}$$

$$AE_{\text{exit}}(2) = AE_{\text{entry}}(2) \cup \{a*b\}$$

$$AE_{\text{exit}}(3) = AE_{\text{entry}}(3) \cup \{a+b\}$$

$$AE_{\text{exit}}(4) = AE_{\text{entry}}(4) \setminus \{a+b, a*b, a-1\}$$


$$AE_{\text{exit}}(5) = AE_{\text{entry}}(5) \cup \{a+b\}$$

Solution of these equations:

$s$	$AE_{\text{entry}}(s)$	$AE_{\text{exit}}(s)$
1	$\emptyset$	$\{a+b\}$
2	$\{a+b\}$	$\{a+b, a*b\}$
3	$\{a+b\}$	$\{a+b\}$
4	$\{a+b\}$	$\emptyset$
5	$\emptyset$	$\{a+b\}$

# Outline

---

- **First example: Available expressions**
- **Basic principles** 
- **More examples**
- **Solving data flow problems**
- **Inter-procedural analysis**
- **Sensitivities**

# Quiz

---

```
var m = x - y;  
if(random()) {  
    while (m > 0) {  
        x = y + 1;  
    }  
}else{  
    n = x - y;  
}  
z = x - y;
```



# Quiz

---

```
var m = x - y;  
if(random()) {  
    while (m > 0) {  
        x = y + 1;  
    }  
}else{  
    n = x - y;  
}
```

```
z = x - y;
```

Is  $x - y$  an available expression when entering this statement?

# Quiz

---

```
var m = x - y;  
if(random()) {  
    while (m > 0) {  
        x = y + 1;  
    }  
} else {  
    n = x - y;  
}  
z = x - y;
```

No, because  
modifying  $x$   
kills  $x - y$

Is  $x - y$  an available  
expression when entering  
this statement?

# Defining a Data Flow Analysis

---

**Any data flow analysis:**  
**Defined by six properties**

- Domain (**facts**)
- Direction
- Transfer function
- Meet operator
- Boundary condition
- Initial values

# Domain

---

- **Analysis associates some information with every program point**
  - Q “Information” means **elements of a set**
- **Domain of the analysis: All possible elements the set may have**
  - Q E.g., for available expressions analysis:  
Domain is set of non-trivial expressions

# Direction

---

- **Analysis propagates information along the control flow graph**
  - Q Forward analysis: Normal flow of control
  - Q Backward analysis: Invert all edges
    - Reasons about executions in reverse
- **E.g., available expression analysis: Forward**

# Transfer Function

---

- Defines how a **statement affects the propagated information**
- $DF_{exit}(s) = \text{some function of } DF_{entry}(s)$
- **E.g., for available expression analysis:**  
 $AE_{exit}(s) = (AE_{entry}(s) \setminus kill(s)) \cup gen(s)$

# Meet Operator

---

- What if **two statements**  $s_1, s_2$  **flow to a statement**  $s$ ?
  - Q Forward analysis: Execution branches merge
  - Q Backward analysis: Branching point
- **Meet operator defines how to combine the incoming information**
  - Q Union:  $DF_{entry}(s) = DF_{exit}(s_1) \cup DF_{exit}(s_2)$
  - Q Intersection:  $DF_{entry}(s) = DF_{exit}(s_1) \cap DF_{exit}(s_2)$

# Meet Operator

---

- What if **two statements**  $s_1, s_2$  **flow to a statement**  $s$ ?
  - Q Forward analysis: Execution branches merge
  - Q Backward analysis: Branching point
- **Meet operator defines how to combine the incoming information**
  - Q Union:  $DF_{entry}(s) = DF_{exit}(s_1) \cup DF_{exit}(s_2)$
  - Q Intersection:  $DF_{entry}(s) = DF_{exit}(s_1) \cap DF_{exit}(s_2)$

 E.g., available expressions analysis



# Boundary Condition

---

- **What information to start with at the first CFG node?**
  - Q Forward analysis: First node is entry node
  - Q Backward analysis: First node is exit node
- **Common choices**
  - Q Empty set
  - Q Entire domain

# Boundary Condition

---

- **What information to start with at the first CFG node?**

- Q Forward analysis: First node is entry node
- Q Backward analysis: First node is exit node

- **Common choices**

- Q Empty set
- Q Entire domain

**E.g., available expressions analysis**

# Initial Values

---

- What is the **information to start with** at **intermediate nodes**?
- **Common choices**
  - Q Empty set
  - Q Entire domain

# Initial Values

---

- What is the **information to start with** at **intermediate nodes**?
- **Common choices**



- Empty set

- Entire domain

**E.g., available expressions analysis**

# Defining a Data Flow Analysis

---

**Any data flow analysis:  
Defined by six properties**

- Domain
- Direction
- Transfer function
- Meet operator
- Boundary condition
- Initial values

# Defining a Data Flow Analysis

---

**Any data flow analysis:  
Defined by six properties**

- Domain
- Direction
- Transfer function
- Meet operator
- Boundary condition
- Initial values
- Non-trivial expressions
- Forward
- $AE_{exit}(s) = (AE_{entry} \setminus kill(s)) \cup gen(s)$
- Intersection ( $\cap$ )
- $AE_{entry}(entryNode) = \emptyset$
- $\emptyset$

**Example: Available expressions** 25 - 2


# Classifying Data Flow Analyses

---

- *Forward* or *backward*, or sometimes both
- Whether facts *may* or *must* be true
  - In a *may* analysis, we care about the facts that *may be true* at *p*. That is, they are true *for some path up to or from p*, depending on the direction of the analysis.
  - In a *must* analysis, we care about the facts that *must be true at p*. That is, they are true *for every path up to or from p*.
- Within a procedure (*intra-procedural*) or between procedures (*inter-procedural*)

# Outline

---

- **First example: Available expressions**
- **Basic principles**
- **More examples** 
- **Solving data flow problems**
- **Inter-procedural analysis**
- **Sensitivities**



# Data Flow Analyses

---

- **Seen previously**

- Q Available expressions

- **Next**

- Q Reaching definitions

- Q Very busy expressions

- Q Live variables